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# Quantum Corrections to the Reissner-Nordström and Kerr-Newman Metrics: Spin 1

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**Abstract**

A previous evaluation of one-photon loop corrections to the energy-momentum tensor has been extended to particles with unit spin and speculations are presented concerning general properties of such forms.

# 1 Introduction

In an earlier letter, we described calculations of the one-loop radiative corrections to the energy-momentum tensor of a charged spinless or spin 1/2 particle of mass $m$ and focused on the nonanalytic component of such results[1]. This is because such nonanalytic pieces involve terms with singularities at small momentum transfer q which, when Fourier-transformed, yield—via the Einstein equations—large distance corrections to the metric tensor. In particular, for both the spinless and spin 1/2 field case, the diagonal component of the metric was shown to be modified from its simple Schwarzschild form via terms which account for the feature that the particle is charged—specifically, in harmonic gauge the resulting metric has the form

$$\begin{aligned} g_{00} &= 1 - \frac{2Gm}{r} + \frac{G\alpha}{r^2} - \frac{8G\hbar}{3\pi m r^3} + \ldots \\ g_{ij} &= -\delta_{ij} - \delta_{ij}\frac{2Gm}{r} + G\alpha\frac{r_i r_j}{r^4} + \frac{4}{3\pi}\frac{G\alpha\hbar}{mr^3}\left(\frac{r_i r_j}{r^2} - \delta_{ij}\right) \\ &- \frac{4}{3\pi}\frac{G\alpha\hbar(1-\log\mu r)}{mr^3}\left(\delta_{ij} - 3\frac{r_i r_j}{r^2}\right) \end{aligned} \tag{1}$$

where $G$ is the gravitational constant and $\alpha = e^2/4\pi$ is the fine structure constant. (Note that the dependence on the arbitrary scale factor $\mu$ can be removed by a coordinate transformation.) The classical—$\hbar$-independent—pieces of these $\alpha$-dependent modifications are well known and can be found by expanding the familiar Reissner-Nordström metric, describing spacetime around a charged, massive object[2]. On the other hand, the calculation also yields quantum mechanical—$\hbar$-dependent—pieces which are new and whose origin can be understood qualitatively as arising from zitterbewegung[1].

In the case of a spin 1/2 system there exists, in addition to the above, a nonvanishing off-diagonal component of the metric, whose radiative corrected form, in harmonic gauge, is

$$g_{0i} = (\vec{S}\times\vec{r})_i\left(\frac{2G}{r^3} - \frac{G\alpha}{mr^4} + \frac{2G\alpha\hbar}{\pi m^2 r^5} + \ldots\right) \tag{2}$$

Here the classical component of this modification can be found by expanding the Kerr-Newman metric[3], which describes spacetime in the vicinity of a charged, massive, and spinning particle, and again there exist quantum corrections due to zitterbewegung[1].

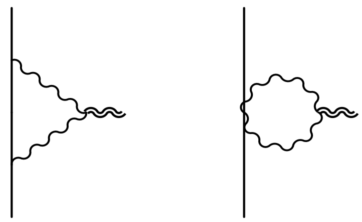


Figure 1: Feynman diagrams having nonanalytic components. Here the single wiggly lines represent photons while the double wiggly line indicates coupling to a graviton.

Based on the feature that the diagonal components were found to have an identical form for both spin 0 and 1/2, it is tempting to speculate that the leading diagonal piece of the metric about a charged particle has a universal form—independent of spin. Whether the same is true for the leading off-diagonal—spin-dependent—component cannot be determined from a single calculation, but it is reasonable to speculate that this is also the case. However, whether these assertions are generally valid can be found only by additional calculation, which is the purpose of the present note, wherein we evaluate the nonanalytic piece of the radiatively-corrected energy-momentum tensor for a charged spin 1 particle—the $W^\pm$ boson—and assess the correctness of this proposal. In the next section then we briefly review the results of the previous paper, and follow with a discussion wherein the calculations are extended to the spin 1 system. Results are summarized in a brief concluding section.

# 2 Lightning Review

Since it important to the remainder of this note, we first present a concise review of the results obtained in our previous paper[1]. In the case of a spin 0 system, the general form of the energy-momentum tensor is

$$< p_2|T_{\mu\nu}(x)|p_1 >_{S=0}= \frac{e^{i(p_2-p_1)\cdot x}}{\sqrt{4E_2E_1}}\left[2P_\mu P_\nu F_1^{(S=0)}(q^2)) + (q_\mu q_\nu - q^2\eta_{\mu\nu})F_2^{(S=0)}(q^2)\right] \tag{3}$$

where $P = \frac{1}{2}(p_1 + p_2)$ is the average momentum, while $q = p_1 - p_2$ is the momentum transfer. The tree level values for these form factors are

$$F_{1,tree}^{(S=0)} = 1 \qquad F_{2,tree}^{(S=0)} = -\frac{1}{2} \tag{4}$$

while the leading nonanalytic loop corrections from Figure 1a and Figure 1b were determined to be

$$\begin{aligned} F_{1,loop}^{(S=0)}(q^2) &= \frac{\alpha}{16\pi}\frac{q^2}{m^2}(8L + 3S) \\ F_{2,loop}^{(S=0)}(q^2) &= \frac{\alpha}{24\pi}(8L + 3S) \end{aligned} \tag{5}$$

where we have defined

$$L = \log(\frac{-q^2}{m^2}) \quad \text{and} \quad S = \pi^2\sqrt{\frac{m^2}{-q^2}}.$$

Such nonanalytic forms, which are singular in the small-q limit, are present due to the presence of *two* massless propagators in the Feynman diagrams[4] and can arise even in electromagnetic diagrams when this situation exists[5]. Upon Fourier-transforming, the piece proportional to $S$ is found to give classical ($\hbar$-independent) behavior while the term involving $L$ is found to yield quantum mechanical ($\hbar$-dependent) corrections.[1] The feature that the form factor $F_1^{(S=0)}(q^2 = 0)$ remains unity even when electromagnetic corrections are included arises from the stricture of energy-momentum conservation[1]. There exists no restriction on $F_2^{(S=0)}(q^2 = 0)$.

In the case of spin 1/2 there exists an additional form factor—$F_3^{(S=\frac{1}{2})}(q^2)$—associated with the existence of spin—

$$\begin{aligned} < p_2|T_{\mu\nu}(x)|p_1 >_{S=\frac{1}{2}} &= \frac{e^{i(p_2-p_1)\cdot x}}{\sqrt{E_1E_2}}\bar{u}(p_2)\left[P_\mu P_\nu F_1^{(S=\frac{1}{2})}(q^2)\right. \\ &+ \frac{1}{2}(q_\mu q_\nu - q^2\eta_{\mu\nu})F_2^{(S=\frac{1}{2})}(q^2) \\ &- \left.\left(\frac{i}{4}\sigma_{\mu\lambda}q^\lambda P_\nu + \frac{i}{4}\sigma_{\nu\lambda}q^\lambda P_\mu\right)F_3^{(S=\frac{1}{2})}(q^2)\right]u(p_1) \end{aligned} \tag{6}$$

[1]The at first surprising feature that a loop calculation can yield classical physics is explained in ref. [4]

The tree level values for these form factors are

$$F_{1,tree}^{(S=\frac{1}{2})} = F_{3,tree}^{(S=\frac{1}{2})} = 1 \qquad F_{2,tree}^{(S=\frac{1}{2})} = 0 \tag{7}$$

while the nonanalytic loop corrections from Figure 1a and Figure 1b were found to be

$$\begin{aligned}
F_{1,loop}^{(S=\frac{1}{2})}(q^2) &= \frac{\alpha}{16\pi}\frac{q^2}{m^2}(8L+3S) \\
F_{2,loop}^{(S=\frac{1}{2})}(q^2) &= \frac{\alpha}{24\pi}(8L+3S) \\
F_{3,loop}^{(S=\frac{1}{2})}(q^2) &= \frac{\alpha}{24\pi}\frac{q^2}{m^2}(4L+3S)
\end{aligned} \tag{8}$$

In this case both $F_1^{(S=\frac{1}{2})}(q^2=0)$ *and* $F_3^{(S=\frac{1}{2})}(q^2=0)$ retain their value of unity even in the presence of electromagnetic corrections. That this must be true for $F_1^{(S=\frac{1}{2})}(q^2=0)$ follows from energy-momentum conservation, as before, while the nonrenormalization of $F_3^{(S=\frac{1}{2})}(q^2=0)$ is required by angular-momentum conservation[6] and asserts that an *anomalous* gravitomagnetic moment is forbidden. The universality of these radiatively corrected forms is suggested by the result

$$F_{1,loop}^{(S=0)}(q^2) = F_{1,loop}^{(S=\frac{1}{2})}(q^2) \quad \text{and} \quad F_{2,loop}^{(S=0)}(q^2) = F_{2,loop}^{(S=\frac{1}{2})}(q^2) \tag{9}$$

but, of course, the spin-dependent gravitomagnetic form factor $F_3^{(S=\frac{1}{2})}(q^2)$ has no analog in the spin 0 sector.

The connection with the metric tensor described in the introduction arises when these results for the energy-momentum tensor are combined with the (linearized) Einstein equation[7]

$$\Box h_{\mu\nu} = -16\pi G\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right) \tag{10}$$

where we have defined

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{11}$$

and

$$T \equiv \mathrm{Tr}\, T_{\mu\nu} \tag{12}$$

Taking Fourier transforms, we find—for both spin 0 and spin 1/2—the diagonal components

$$\begin{aligned} h_{00}(\vec{r}) &= -16\pi G \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \frac{1}{\vec{k}^2} \left( \frac{m}{2} - \frac{\alpha\pi|\vec{k}|}{8} - \frac{\alpha\vec{k}^2}{3\pi m} \log\frac{\vec{k}^2}{m^2} \right) \\ h_{ij}(\vec{r}) &= -16\pi G \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \frac{1}{\vec{k}^2} \left( \frac{m}{2}\delta_{ij} + (\frac{\alpha\pi}{16|\vec{k}|} + \frac{\alpha}{6\pi m} \log\frac{\vec{k}^2}{m^2})(k_ik_j - \delta_{ij}\vec{k}^2) \right) \end{aligned} \tag{13}$$

while from the spin 1/2 gravitomagnetic form factor we find the off-diagonal term

$$h_{0i}(\vec{r}) = -16\pi G i \int \frac{d^3k}{(2\pi)^3} \frac{1}{\vec{k}^2} \left( \frac{1}{2} - \frac{\alpha\pi|\vec{k}|}{16m} - \frac{\alpha\vec{k}^2}{12\pi m^2} \log\frac{\vec{k}^2}{m^2} \right) (\vec{S}\times\vec{k})_i \tag{14}$$

Evaluating the various Fourier transforms, we find the results quoted in the introduction[8].

The purpose of the present note is to study how these results generalize to the case of higher spin. Specifically, we shall below examine the radiative corrections to the energy-momentum tensor of a charged spin 1 system.

# 3 Spin 1

A neutral spin 1 field $\phi_\mu(x)$ having mass $m$ is described by the Proca Lagrangian, which is of the form[9]

$$\mathcal{L}(x) = -\frac{1}{4}U_{\mu\nu}(x)U^{\mu\nu}(x) + \frac{1}{2}m^2\phi_\mu(x)\phi^\mu(x) \tag{15}$$

where

$$U_{\mu\nu}(x) = i\partial_\mu\phi_\nu(x) - i\partial_\nu\phi_\mu(x) \tag{16}$$

is the spin 1 field tensor. If the particle has charge $e$, we can generate a gauge-invariant form of Eq. 15 by use of the well-known minimal substitution[10]—defining

$$\pi_\mu = i\partial_\mu - eA_\mu(x) \tag{17}$$

and

$$U_{\mu\nu}(x) = \pi_\mu\phi_\nu(x) - \pi_\nu\phi_\mu(x) \tag{18}$$

the charged Proca Lagrangian density becomes

$$\mathcal{L}(x) = -\frac{1}{2}U^\dagger_{\mu\nu}(x)U^{\mu\nu}(x) + m^2\phi^\dagger_\mu(x)\phi^\mu(x) \tag{19}$$

Introducing the left-right derivative

$$D(x)\overleftrightarrow{\nabla}F(x) \equiv D(x)\nabla F(x) - (\nabla D(x))F(x) \tag{20}$$

the single-photon component of the interaction can be written as

$$\mathcal{L}_{int}(x) = ieA^\mu(x)\phi^{\alpha\dagger}(x)[\eta_{\alpha\beta}\overleftrightarrow{\nabla}_\mu - \eta_{\beta\mu}\nabla_\alpha]\phi^\beta(x) + \eta_{\alpha\mu}(\nabla_\beta\phi^{\alpha\dagger}(x))\phi^\beta(x) \tag{21}$$

so that the on-shell matrix element of the electromagnetic current becomes

$$\frac{1}{\sqrt{4E_fE_i}} < p_f, \epsilon_B|j_\mu|p_i, \epsilon_A >= -\frac{e}{\sqrt{4E_fE_i}}\left[2P_\mu\epsilon^*_B\cdot\epsilon_A - \epsilon_{A\mu}\epsilon^*_B\cdot q + \epsilon^*_{B\mu}\epsilon_A\cdot q\right] \tag{22}$$

where we have used the property $p_f\cdot\epsilon^*_B = p_i\cdot\epsilon_A = 0$ for the Proca polarization vectors. If we now look at the spatial piece of this term we find

$$\frac{1}{\sqrt{4E_fE_i}} < p_f, \epsilon_B|\vec{\epsilon_\gamma}\cdot\vec{j}|p_i, \epsilon_A >\simeq \frac{e}{2m}\vec{\epsilon}_\gamma\times\vec{q}\cdot\hat{\epsilon}^*_B\times\hat{\epsilon_A} = \frac{e}{2m} < 1, m_f|\vec{S}|1, m_i > \cdot\vec{B} \tag{23}$$

where we have used the result that in the Breit frame for a nonrelativistically moving particle

$$i\hat{\epsilon}^*_B\times\hat{\epsilon}_A =< 1, m_f|\vec{S}|1, m_i > \tag{24}$$

which we recognize as representing a magnetic moment interaction with g=1. On the other hand if we take the time component of Eq. 22, we find, again in the Breit frame and a nonrelativistically moving system

$$\frac{1}{\sqrt{4E_fE_i}} < p_f, \epsilon_B|\epsilon_{0\gamma}j_0|p_i, \epsilon_A >\simeq -e\epsilon_{0\gamma}\left[\epsilon^*_B\cdot\epsilon_A + \frac{1}{2m}(\epsilon_{A0}\hat{\epsilon}^*_B\cdot\vec{q} - \epsilon^*_{B0}\hat{\epsilon}_A\cdot\vec{q})\right] \tag{25}$$

Using

$$\begin{aligned}\epsilon^0_A \simeq \frac{1}{2m}\hat{\epsilon}_A\cdot\vec{q}, \qquad \epsilon^0_B \simeq -\frac{1}{2m}\hat{\epsilon}^*_B\cdot\vec{q}\\ \epsilon^*_B\cdot\epsilon_A \simeq -\hat{\epsilon}^*_B\cdot\hat{\epsilon}_A - \frac{1}{2m^2}\hat{\epsilon}^*_B\cdot\vec{q}\hat{\epsilon}_A\cdot\vec{q}\end{aligned} \tag{26}$$

we observe that

$$\frac{1}{\sqrt{4E_fE_i}} < p_f, \epsilon_B|\epsilon_{0\gamma}j_0|p_i, \epsilon_A > \simeq e\epsilon_{0\gamma}\hat{\epsilon}_B^* \cdot \hat{\epsilon}_A \tag{27}$$

which is the expected electric monopole term—any electric quadrupole contributions have cancelled[11]. Overall then, Eq. 22 corresponds to a simple E0 interaction with the charge accompanied by an M1 interaction with g-factor unity, which is consistent with the speculation by Belinfante that for a particle of spin $S$, $g = 1/S$[12].

Despite this suggestively simple result, however, Eq. 15 does *not* correctly describe the interaction of the charged $W$-boson field, due to the feature that the $W^\pm$ are components of an SU(2) vector field[13]. The proper Proca Lagrangian has the form

$$\mathcal{L}(x) = -\frac{1}{4}\vec{U}^\dagger_{\mu\nu}(x) \cdot \vec{U}^{\mu\nu}(x) + \frac{1}{2}m_W^2\vec{\phi}_\mu(x) \cdot \vec{\phi}^\mu(x) \tag{28}$$

where the field tensor $\vec{U}_{\mu\nu}(x)$ contains an additional term on account of gauge invariance

$$\vec{U}_{\mu\nu}(x) = \pi_\mu\vec{U}_\nu(x) - \pi_\nu\vec{U}_\mu(x) - ig\vec{U}_\mu(x) \times \vec{U}_\nu(x) \tag{29}$$

with $g$ being the SU(2) electroweak coupling coupling constant. The Lagrange density Eq. 28 then contains the piece

$$\mathcal{L}_{int}(x) = -gW^{0\mu\nu}(x)(W_\mu^{+\dagger}(x)W_\nu^+(x) - W_\mu^{-\dagger}(x)W_\mu^-)(x) \tag{30}$$

among (many) others. However, in the standard model the neutral member of the W-triplet is a linear combination of $Z^0$ and photon fields[14]—

$$W_\mu^0 = \cos\theta_W Z_\mu^0 + \sin\theta_W A_\mu \tag{31}$$

and, since $g\sin\theta_W = e$, we have a term in the interaction Lagrangian

$$\mathcal{L}_{int}^{(1)}(x) = -eF^{\mu\nu}(x)(W_\mu^{+\dagger}(x)W_\nu^+(x) - W_\mu^{-\dagger}(x)W_\mu^-(x)) \tag{32}$$

which represents an additional interaction that must be appended to the convention Proca result. In the Breit frame and for a nonrelativistically moving system we have

$$\frac{1}{\sqrt{4E_fE_i}} < p_f, \epsilon_B|\vec{\epsilon_\gamma}\cdot\vec{j}^{(1)}|p_i, \epsilon_A > \simeq \frac{e}{2m_W}\vec{\epsilon}_\gamma\times\vec{q}\cdot\hat{\epsilon}_B^*\times\hat{\epsilon_A} = \frac{e}{2m_W} < 1, m_f|\vec{S}|1, m_i > \cdot\vec{B} \tag{33}$$

and

$$\frac{1}{\sqrt{4E_fE_i}} < p_f, \epsilon_B | j_0^{(1)} | p_i, \epsilon_A > \simeq -e\frac{1}{2m_W}(\epsilon_A^0 \hat{\epsilon}_B^* \cdot \vec{q} - \epsilon_{B0}^* \hat{\epsilon}_A \cdot \vec{q}) = -\frac{e}{2m_W^2}\hat{\epsilon}_B^* \cdot \vec{q}\hat{\epsilon}_A \cdot \vec{q} \tag{34}$$

The first piece—Eq. 33—constitutes an additional magnetic moment and modifies the W-boson g-factor from its Belinfante value of unity to its standard model value of 2. Using

$$\frac{1}{2}(\epsilon_{Bi}^*\epsilon_{Aj} + \epsilon_{Ai}\epsilon_{Bj}^*) - \frac{1}{3}\delta_{ij}\hat{\epsilon}_B^* \cdot \hat{\epsilon}_A =< 1, m_f | \frac{1}{2}(S_iS_j + S_jS_i) - \frac{2}{3}\delta_{ij} | 1, m_i > \tag{35}$$

we observe that the second component—Eq. 34—implies the existence of a quadrupole moment of size $Q = -e/M_W^2$. Both of these results are well known predictions of the standard model for the charged vector bosons[15].

In fact, it has recently been argued, from a number of viewpoints, that the "natural" value of the gyromagnetic ratio for a particle of *arbitrary* spin should be g=2[16], as opposed to the value $1/S$ from the Belinfante conjecture, and we shall consequently employ g=2 in our spin 1 calculations below.

Having obtained the appropriate Langrangian for the interactions of a charged spin-1 system,

$$\mathcal{L}(x) = -\frac{1}{2}U_{\mu\nu}^\dagger(x)U^{\mu\nu}(x) + m^2\phi_\mu^\dagger(x)\phi^\mu(x) - eF^{\mu\nu}(x)(\phi_\mu^{+\dagger}(x)\phi_\nu^+(x) - \phi_\mu^{-\dagger}(x)\phi_\mu^-(x)) \tag{36}$$

we can now calculate the matrix elements which will be needed in our calculation. Specifically, the general single photon vertex for a transition involving an outgoing photon with polarization index $\mu$ and four-momentum $q = p1 - p2$, an incoming spin one particle with polarization index $\alpha$ and four-momentum $p_1$ together with an outgoing spin one particle with polarization index $\beta$ and four-momentum $p_2$ is[17]

$$V_{\beta,\alpha,\mu}^{(1)}(p_1, p_2) = -ie\left[(p_1 + p_2)_\mu\eta_{\alpha\beta} - (gp_{1\beta} - (g-1)p_{2\beta})\eta_{\alpha\mu} - (gp_{2\alpha} - (g-1)p_{1\alpha})\eta_{\beta\mu})\right] \tag{37}$$

while the two-photon vertex with polarization indices $\mu, \nu$, an incoming spin one particle with polarization index $\alpha$ and four-momentum $p_1$ together with an outgoing spin one particle with polarization index $\beta$ and four-momentum $p_2$ has the form[18]

$$V_{\beta,\alpha,\mu\nu}^{(2)}(p_1, p_2) = -ie^2(2\eta_{\mu\nu}\eta_{\alpha\beta} - \eta_{\alpha\mu}\eta_{\beta\nu} - \eta_{\alpha\nu}\eta_{\beta\mu}) \tag{38}$$

The energy-momentum tensor connecting an incoming vector meson with polarization index $\alpha$ and four-momentum $k_1$ with and outgoing vector with polarization index $\beta$ and four-momentum $k_2$ is found to be[19]

$$\begin{aligned} < k_2,\beta|T_{\mu\nu}|k_1,\alpha > &= (k_{1\mu}k_{2\nu}+k_{1\nu}k_{2\mu})\eta_{\alpha\beta} \\ &- k_{1\beta}(k_{2\mu}\eta_{\alpha\nu}+k_{2\nu}\eta_{\alpha\mu}) \\ &- k_{2\alpha}(k_{1\nu}\eta_{\beta\mu}+k_{1\mu}\eta_{\beta\nu}) \\ &+ (k_1\cdot k_2-m^2)(\eta_{\beta\mu}\eta_{\alpha\nu}+\eta_{\beta\nu}\eta_{\alpha\mu}) \\ &- \eta_{\mu\nu}[(k_1\cdot k_2-m^2)\eta_{\alpha\beta}-k_{1\beta}k_{2\alpha}] \end{aligned} \tag{39}$$

and that between photon states is identical, except that the terms in $m^2$ are absent. For later use, it is useful to note that the trace of this expression has the simple form

$$\eta^{\mu\nu} < k_2,\beta|T_{\mu\nu}|k_1,\alpha >= 2m^2\eta_{\alpha\beta} \tag{40}$$

which vanishes in the case of the photon. The leading component of the on-shell energy-momentum tensor between charged vector meson states is then

$$\begin{aligned} < k_2,\epsilon_B|T^{(0)}_{\mu\nu}|k_1,\epsilon_A > &= (k_{1\mu}k_{2\nu}+k_{1\nu}k_{2\mu})\epsilon^*_B\cdot\epsilon_A \\ &- k_1\cdot\epsilon^*_B(k_{2\mu}\epsilon_{A\nu}+k_{2\nu}\epsilon_{A\mu} \\ &- k_2\cdot\epsilon_A(k_{1\nu}\eta^*_{B\mu}+k_{1\mu}\epsilon^*_{B\nu}) \\ &+ (k_1\cdot k_2-m^2)(\epsilon^*_{B\mu}\epsilon_{A\nu}+\epsilon^*_{B\nu}\epsilon_{A\mu}) \\ &- \eta_{\mu\nu}[(k1\cdot k_2-m^2)\epsilon^*_B\cdot\epsilon_A-k_1\cdot\epsilon^*_Bk_2\cdot\epsilon_A] \end{aligned} \tag{41}$$

and the focus of our calculation is to evaluate the one-loop electromagnetic corrections to Eq. 41, via the diagrams shown in Figure 1, keeping only the leading nonanalytic terms. Details of the calculation are described in the appendix, and the results are given by

a) Seagull loop diagram (Figure 1a)

$$\begin{aligned} Amp[a]_{\mu\nu} &= \frac{L\alpha}{48\pi m}\Big[\epsilon_A\cdot\epsilon^*_B(2q_\mu q_\nu+\frac{1}{2}q^2\eta_{\mu\nu})-\epsilon_A\cdot q\epsilon^*_B\cdot q\eta_{\mu\nu} \\ &+ \epsilon_A\cdot q(\epsilon^*_{B\mu}q_\nu+\epsilon^*_{B\nu}q_\mu)+\epsilon^*_B\cdot q(\epsilon_{A\mu}q_\nu+\epsilon_{A\nu}q_\mu) \\ &- 2(\epsilon_{A\mu}\epsilon^*_{B\nu}+\epsilon_{A\nu}\epsilon^*_{B\mu})q^2\Big] \end{aligned} \tag{42}$$

b) Born loop diagram (Figure 1b)

$$\begin{aligned} Amp[b]_{\mu\nu} &= \frac{\alpha}{48\pi m}\left\{-3P_\mu P_\nu q^2\epsilon_B^*\cdot\epsilon_A(8L+3S)\right.\\ &+ [(P_\mu\epsilon_{A\nu}+P_\nu\epsilon_{A\mu})\epsilon_B^*\cdot q-(P_\mu\epsilon_{B\nu}^*+P_\nu\epsilon_{B\mu}^*\epsilon_A\cdot q)]q^2(4L+3S)\\ &- [\epsilon_A\cdot q(\epsilon_{B\mu}^*q_\nu+\epsilon_{B\nu}^*q_\mu)+\epsilon_B^*\cdot q(\epsilon_{A\mu}q_\nu+\epsilon_{A\nu}q_\mu)]L\\ &- [q_\mu q_\nu(10L+3S)-q^2\eta_{\mu\nu}(\frac{15}{2}L+3S)]\epsilon_B^*\cdot\epsilon_A\\ &+ 2(\epsilon_{B\mu}^*\epsilon_{A\nu}+\epsilon_{B\nu}^*\epsilon_{A\mu})q^2L\\ &+ \left.\ldots\right\} \end{aligned} \tag{43}$$

Note that due to conservation of the energy-momentum tensor—$\partial^\mu T_{\mu\nu} = 0$—the on-shell matrix element must satisfy the gauge invariance condition

$$q^\nu < k_2, \epsilon_B|T_{\mu\nu}|k_1, \epsilon_A >= 0$$

In our case, although the leading order contribution satisfies this condition

$$q^\mu < k_2, \epsilon_B|T_{\mu\nu}^{(0)}|k_1, \epsilon_A >= 0 \tag{44}$$

the contribution from *neither* diagram 1a or 1b is gauge-invariant

$$\begin{aligned} q^\mu Amp[a]_{\mu\nu} &= \frac{\alpha L}{96\pi m}\left[5q^2q_\nu\epsilon_B^*\cdot\epsilon_A-2q^2(\epsilon_{A\nu}\epsilon_B^*\cdot q+\epsilon_{B\nu}^*\epsilon_A\cdot q))+2q_\mu\epsilon_B^*\cdot q\epsilon_a\cdot q\right]\\ q^\mu Amp[b]_{\mu\nu} &= -\frac{\alpha L}{96\pi m}\left[5q^2q_\nu\epsilon_B^*\cdot\epsilon_A-2q^2(\epsilon_{A\nu}\epsilon_B^*\cdot q+\epsilon_{B\nu}^*\epsilon_A\cdot q))+2q_\mu\epsilon_B^*\cdot q\epsilon_a\cdot q\right] \end{aligned} \tag{45}$$

Nevertheless the sum of these terms *is* gauge-invariant—

$$q^\mu(Amp[a]_{\mu\nu}+Amp[b]_{\mu\nu}) = 0 \tag{46}$$

which serves as an important check on our result.

The full loop contribution is then

$$\begin{aligned} Amp[a+b]_{\mu\nu} &= \frac{\alpha}{48\pi m}\left\{\epsilon_B^*\cdot\epsilon_A[(q_\mu q_\nu-q^2\eta_{\mu\nu})-3P_\mu P_\nu q^2](8L+3S)\right.\\ &+ ((P_\mu\epsilon_{A\nu}+P_\nu\epsilon_{A\mu})\epsilon_B^*\cdot q-(P_\mu\epsilon_{B\nu}^*+P_\nu\epsilon_{B\mu}^*)\epsilon_A\cdot q)q^2(4L+3S)\\ &+ \left.\ldots\right\} \end{aligned} \tag{47}$$

Due to covariance and gauge invariance the form of the matrix element of $T_{\mu\nu}$ between on-shell spin 1 states must be expressible in the form

$$\begin{aligned}
&& < p_2, \epsilon_B | T_{\mu\nu}(x) | p_1, \epsilon_A >_{S=1} = -\frac{e^{i(p_2-p_1)\cdot x}}{\sqrt{4E_1E_2}}[2P_\mu P_\nu \epsilon_B^* \cdot \epsilon_A F_1(^{(S=1)} q^2) \\
&+& (q_\mu q_\nu - \eta_{\mu\nu} q^2)\epsilon_B^* \cdot \epsilon_A F_2^{(S=1)}(q^2) \\
&+& [P_\mu(\epsilon_{B\nu}^* \epsilon_A \cdot q - \epsilon_{A\nu}\epsilon_B^* \cdot q) + P_\nu(\epsilon_{B\mu}^* \epsilon_A \cdot q - \epsilon_{A\mu}\epsilon_B^* \cdot q)]F_3^{(S=1)}(q^2) \\
&+& \big[(\epsilon_{A\mu}\epsilon_{B\nu}^* + \epsilon_{B\mu}^* \epsilon_{A\nu})q^2 - (\epsilon_{B\mu}^* q_\nu + \epsilon_{B\nu}^* q_\mu)\epsilon_A \cdot q \\
&-& (\epsilon_{A\mu} q_\nu + \epsilon_{A\nu} q_\mu)\epsilon_B^* \cdot q + 2\eta_{\mu\nu}\epsilon_A \cdot q \epsilon_B^* \cdot q\big]\, F_4^{(S=1)}(q^2) \\
&+& \frac{2}{m^2} P_\mu P_\nu \epsilon_A \cdot q \epsilon_B^* \cdot q F_5^{(S=1)}(q^2) \\
&+& \frac{1}{m^2}(q_\mu q_\nu - \eta_{\mu\nu} q^2)\epsilon_A \cdot q \epsilon_B \cdot q F_6^{(S=1)}(q^2)] \qquad (48)
\end{aligned}$$

Here $F_1^{(S=1)}(q^2), F_2^{(S=1)}(q^2), F_3^{(S=1)}(q^2)$ correspond to their spin 1/2 counterparts $F_1^{(S=\frac{1}{2})}(q^2), F_2^{(S=\frac{1}{2})}(q^2), F_3^{(S=\frac{1}{2})}(q^2)$, while $F_4^{(S=1)}(q^2), F_5^{(S=1)}(q^2), F_6^{(S=1)}(q^2)$ represent new forms unique to spin 1. (Note that each kinematic form in Eq. 48 is separately gauge invariant.)

The results of the calculation described above can most concisely be described in terms of these form factors. Thus the tree level predictions can be described as

$$\begin{aligned}
F_{1,tree}^{(S=1)} &=& F_{3,tree}^{(S=1)} = 1 \\
F_{2,tree}^{(S=1)} &=& F_{4,tree}^{(S=1)} = -\frac{1}{2} \\
F_{5,tree}^{(S=1)} &=& F_{6,tree}^{(S=1)} = 0 \qquad (49)
\end{aligned}$$

while the full radiatively corrected values are given by

$$\begin{aligned}
F_1^{(S=1)}(q^2) &= 1+\frac{\alpha}{16\pi}\frac{q^2}{m^2}(8L+3S)+\ldots \\
F_2^{(S=1)}(q^2) &= -\frac{1}{2}+\frac{\alpha}{24\pi}(8L+3S)+\ldots \\
F_3^{(S=1)}(q^2) &= 1+\frac{\alpha}{24\pi}\frac{q^2}{m^2}(4L+3S)+\ldots \\
F_4^{(S=1)}(q^2) &= -\frac{1}{2}+\frac{\alpha}{192\pi}\frac{q^2}{m^2}(16L-3S)+\ldots \\
F_5^{(S=1)}(q^2) &= \frac{\alpha}{384\pi}\frac{q^2}{m^2}(64L+9S)+\ldots \\
F_6^{(S=1)}(q^2) &= \frac{\alpha}{192\pi}(64L+15S)+\ldots
\end{aligned} \tag{50}$$

and we note that $F_{1,2,3,loop}^{(S=1)}(q^2)$ as found for unit spin agree exactly with the forms $F_{1,2,3,loop}^{(S=\frac{1}{2})}(q^2)$found previously for spin 1/2 and with $F_{1,2,loop}^{(S=0)}(q^2)$ in the spinless case. We observe that both $F_1^{(S=1)}(q^2=0)=1$ and $F_3^{(S=1)}(q^2=0)=1$ as required by energy-momentum and angular momentum conservation. Interestingly, the form factors $F_4^{(S=1)}(q^2)$ and $F_5^{(S=1)}(q^2)$ are also unrenormalized from their tree level values and this fact has an interesting consequence. Since, in the Breit frame and using nonrelativistic kinematics we have

$$\begin{aligned}
&< p_2,\epsilon_B|T_{00}(0)|p_1,\epsilon_A > \simeq m\left\{\hat{\epsilon}_B^*\cdot\hat{\epsilon}_A F_1^{(S=1)}(q^2)+\frac{1}{2m^2}\hat{\epsilon}_B^*\cdot\vec{q}\hat{\epsilon}_A\cdot\vec{q}\right. \\
\times\ &\left.[F_1^{(S=1)}(q^2)-F_2^{(S=1)}(q^2)-2(F_4^{(S=1)}(q^2)+F_5^{(S=1)}(q^2)-\frac{q^2}{2m^2}F_6^{(S=1)}(q^2))]\right\}+\ldots \\
&< p_2,\epsilon_B|T_{0i}(0)|p_1,\epsilon_A > \simeq -\frac{1}{2}[(\hat{\epsilon}_B^*\times\hat{\epsilon}_A)\times\vec{q}]_i F_3^{(S-1)}(q^2)+\ldots
\end{aligned} \tag{51}$$

we can identify values for the gravitoelectric monopole, gravitomagnetic dipole, and gravitoelectric quadrupole coupling constants

$$\begin{aligned}
K_{E0} &= mF_1^{(S=1)}(q^2=0) \\
K_{M1} &= \frac{1}{2}F_3^{(S=1)}(q^2=0) \\
K_{E2} &= \frac{1}{2m}\left[F_1^{(S=1)}(q^2=0)-F_3^{(S=1)}(q^2=0)-2F_4^{(S=1)}(q^2=0)-2F_5^{(S=1)}(q^2=0)\right]
\end{aligned} \tag{52}$$

Taking $Q_g \equiv m$ as the gravitational "charge," we observe that the tree level values—

$$K_{E0} = Q_g \qquad K_{M1} = \frac{Q_g}{2m} \qquad K_{E2} = \frac{Q_g}{m^2} \tag{53}$$

are *unrenormalized* by loop corrections. That is to say, not only does there not exist any anomalous gravitomagnetic moment, as mentioned above, but also there is no anomalous gravitoelectric quadrupole moment.

As an aside, we note that an additional check on our results is found by using the feature pointed out above that the radiative correction terms must be traceless. In our case this means that two conditions must be satisfied, since there are two separate scalar structures—$\epsilon_A \cdot \epsilon_B$ and $\epsilon_A \cdot q \epsilon_B \cdot q$—whose coefficients must vanish, which leads to

$$\begin{aligned}
\epsilon_A \cdot \epsilon_B \quad &: \quad P^2 F_1^{(S=1)}(q^2) - \frac{3}{2}\frac{q^2}{m^2} F_2^{(S=1)}(q^2) + 2q^2 F_4^{(S=1)}(q^2) = 0 \\
\epsilon_A \cdot q \epsilon_B \cdot q \quad &: \quad F_3^{(S=1)}(q^2) + 4F_4^{(S=1)}(q^2) + \frac{P^2}{m^2} F_5^{(S=1)}(q^2) - 3\frac{q^2}{m^2} F_6^{(S=1)}(q^2) = 0
\end{aligned} \tag{54}$$

Both strictures are satisfied at the level to which we work.

### 3.1 An Addendum

Before concluding, it is interesting to note that although we have used the standard model value $g = 2$ which is appropriate for a charged W-boson, it is not necessary to do so. Indeed for a charged spin 1 system like the $\rho^\pm$ strong interaction corrections lead to a very different value of the gyromagnetic ratio, and it is interesting to calculate the form factors which would result from an arbitrary choice of $g$. The results are found to be

$$\begin{aligned}
F_1^{(S=1)}(q^2) &= 1+\frac{\alpha}{16\pi}\frac{q^2}{m^2}(8L+3S)+\ldots \\
F_2^{(S=1)}(q^2) &= -\frac{1}{2}+\frac{\alpha}{24\pi}(8L+3S)+\ldots \\
F_3^{(S=1)}(q^2) &= 1+\frac{\alpha}{48\pi}\frac{q^2}{m^2}((-(4+g(g-8))L+3gS)+\ldots \\
F_4^{(S=1)}(q^2) &= -\frac{1}{2}+\frac{\alpha}{1536\pi}\frac{q^2}{m^2}(16(g(3g-4)+4)L-3(g(g+4)-4)S)+\ldots \\
F_5^{(S=1)}(q^2) &= -\frac{\alpha}{3072\pi}\frac{q^2}{m^2}(32(g-6)(g+2)L+3(g(5g-12)-20)S)+\ldots \\
F_6^{(S=1)}(q^2) &= \frac{\alpha}{1536\pi}(32(g+2)^2L+3(g(20-3g)+12)S)+\ldots
\end{aligned} \tag{55}$$

Of course, one easily confirms that in the limit $g \to 2$ these results reproduce those given in Eq. 50. However, there is something else of interest here. In ref. [1] it was shown that the classical piece of the leading form factors follows simply from the requirement that the long range component of the matrix element of the energy-momentum tensor agree with the well known form

$$T_{\mu\nu} = -F_{\mu\lambda}F_\nu{}^\lambda + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \tag{56}$$

For the diagonal components of $T_{\mu\nu}$, this leads for a point charge $e$ to

$$\begin{aligned}
T_{00} &= \frac{1}{2}\vec{E}^2 = \frac{\alpha}{8\pi r^4} \\
T_{0i} &= 0 \\
T_{ij} &= -E_iE_j + \frac{1}{2}\delta_{ij}\vec{E}^2 = -\frac{\alpha}{4\pi r^4}\left(\frac{r_ir_j}{r^2}-\frac{1}{2}\delta_{ij}\right)
\end{aligned} \tag{57}$$

which can easily be shown to correspond to Fourier transform of the leading diagonal classical loop corrections to the energy-momentum tensor, *i.e.,* those terms of the loop calculation proportional to S—

$$\begin{aligned}
T_{00}^{loop,cl}(\vec{r}) &= \int\frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{r}}T_{00}^{loop,cl}(\vec{k}) = \frac{\alpha}{8\pi r^4} \\
T_{0i}^{loop,cl}(\vec{r}) &= 0 \\
T_{ij}^{loop,cl}(\vec{r}) &= \int\frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{r}}T_{ij}^{loop,cl}(\vec{k}) = -\frac{\alpha}{4\pi r^4}\left(\frac{r_ir_j}{r^2}-\frac{1}{2}\delta_{ij}\right)
\end{aligned} \tag{58}$$

where we have used the forms

$$\begin{aligned} T_{00}^{loop,cl}(\vec{k}) &= -\frac{\alpha\pi|\vec{k}|}{8} \\ T_{ij}^{loop,cl}(\vec{k}) &= \frac{\alpha\pi}{16|\vec{k}|}(k_ik_j - \delta_{ij}\vec{k}^2) \end{aligned} \tag{59}$$

found above.

However, when the charged particle carries spin, a magnetic field

$$\vec{B} = \frac{ge}{2m}\frac{3\hat{r}\vec{S}\cdot\hat{r} - \vec{S}}{4\pi r^3} \tag{60}$$

must also be included. This changes the diagonal pieces of the energy-momentum tensor by calculable higher order terms, which means that the *leading* diagonal pieces determined by the form factors $F_1(q^2), F_2(q^2)$ should be unchanged and therefore independent of $g$ as found in Eq. 55. On the other hand the presence of a magnetic field leads to a nonvanishing off-diagonal component

$$T_{0i} = -(\vec{E}\times\vec{B})_i = -\frac{\alpha g}{8\pi m r^6}(\vec{S}\times\vec{r})_i \tag{61}$$

which means that the classical component of our loop correction to the off-diagonal component of the energy momentum tensor must also have this form

$$T_{0i}^{loop,cl}(\vec{r}) = \int\frac{d^3k}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{r}}T_{0i}^{loop,cl}(\vec{k}) \tag{62}$$

From Eq. 55 we find

$$T_{0i}^{loop,cl}(\vec{k}) = -i\frac{\alpha\pi g|\vec{k}|}{32m}(\vec{S}\times\vec{k})_i \tag{63}$$

whose Fourier transform yields the expected form. Thus the linear proportionality of the classical loop correction to $F_3(q^2)$ to the gyromagnetic ratio $g$ is expected, and it is only if the value $g = 2$ is employed that the Kerr-Newman metric is reproduced. Nevertheless, we find that other consistent values to the metric tensor are reproduced if alternative values of the gyromagnetic ratio apply. In the case of the quantum component of the loop correction the ratio of the spin 1 and spin 1/2 terms is found to be

$$\frac{F_3^{loop,qm}}{F_3^{loop,qm}} = g(1-\frac{g}{8}) - \frac{1}{2} \tag{64}$$

so that again use of the value $g = 2$ guarantees universality for both the classical and the quantum correction as well as brings these results into conformity with the Kerr-Newman form. This point deserves further study.

# 4 Conclusion

Above we have calculated the radiative corrections to the energy-momentum tensor of a spin 1 system. We have confirmed the universality which was speculated in our previous work in that we have confirmed that

$$\begin{aligned} F_{1,loop}^{(S=0)}(q^2) &= F_{1,loop}^{(S=\frac{1}{2})}(q^2) = F_{1,loop}^{(S=1)}(q^2) \\ F_{2,loop}^{(S=0)}(q^2) &= F_{2,loop}^{(S=\frac{1}{2})}(q^2) = F_{2,loop}^{(S=1)}(q^2) \\ F_{3,loop}^{(S=\frac{1}{2})}(q^2) &= F_{3,loop}^{(S=1)}(q^2) \end{aligned} \tag{65}$$

The universality in the case of the classical (square root) nonanalyticities is not surprising and in fact is *required* by the connection to the metric tensor and to the classical form of the energy-momentum tensor—Eqn. 56. In the case of the quantum (logarithmic) nonanalyticities, however, it is not clear why these terms must be universal. We also found additional higher order form factors for spin 1, which also receive loop corrections. It is tempting to conclude that this radiative correction universality holds for arbitrary spin. However, it is probably not possible to show this by generalizing the calculations above. Indeed the spin 1 result involves *considerably* more computation than does its spin 1/2 counterpart, which was already much more tedious than that for spin 0. Perhaps a generalization such as that used in nuclear beta decay can be employed[20]. Work is underway on this problem and will be reported in an upcoming communication.

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# 5 Appendix

In this section we sketch how our results were obtained. The basic idea is to calculate the Feynman diagrams shown in Figure 1. Thus for Figure 1a we find[21]

$$Amp[a]_{\mu\nu} = \frac{1}{2!}\int \frac{d^4k}{(2\pi)^4}\frac{\epsilon_B^{*\beta}V^{(2)}_{\beta,\alpha,\mu\nu}(p1,p2)\epsilon_A^{\alpha} < k-q,\beta|T_{\mu\nu}|k,\alpha > |_{m^2=0}}{k^2(k-q)^2} \tag{66}$$

while for Figure 1b[21]

$$\begin{aligned}
Amp[b]_{\mu\nu} &= \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k-q)^2((k-p)^2-m^2)} \\
&\times \epsilon_B^{\beta}V^{(1)}_{\beta,\lambda,\kappa}(p_2,p_1-k)\left(-\eta^{\lambda\zeta}+\frac{(p1-k)^{\lambda}(p_1-k)^{\zeta}}{m^2}\right) \\
&\times V^{(1)}_{\zeta,\rho,\delta}(p1-k,p1)\epsilon_A^{\rho} < k,\kappa|T_{\mu\nu}|k-q,\delta >_{m^2=0}
\end{aligned} \tag{67}$$

Here the various vertex functions are listed in section 3, while for the integrals, all that is needed is the leading nonanalytic behavior. Thus we use

$$\begin{aligned}
I(q) &= \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k-q)^2} = \frac{-i}{32\pi^2}(2L+\ldots) \\
I_\mu(q) &= \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu}{k^2(k-q)^2} = \frac{i}{32\pi^2}(q_\mu L+\ldots) \\
I_{\mu\nu}(q) &= \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{k^2(k-q)^2} = \frac{-i}{32\pi^2}(q_\mu q_\nu\frac{2}{3}L - q^2\eta_{\mu\nu}\frac{1}{6}L+\ldots) \\
I_{\mu\nu\alpha}(q) &= \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu k_\alpha}{k^2(k-q)^2} = \frac{i}{32\pi^2}(-q_\mu q_\nu q_\alpha\frac{L}{2} \\
&+ (\eta_{\mu\nu}q_\alpha+\eta_{\mu\alpha}q_\nu+\eta_{\nu\alpha}q_\mu)\frac{1}{12}Lq^2+\ldots)
\end{aligned} \tag{68}$$

for the "bubble" integrals and

$$
\begin{aligned}
J(p,q) &= \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2(k-q)^2((k-p)^2-m^2)} = \frac{-i}{32\pi^2 m^2}(L+S)+\ldots \\
J_\mu(p,q) &= \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu}{k^2(k-q)^2((k-p)^2-m^2)} = \frac{i}{32\pi^2 m^2} \\
&\times \ [p_\mu((1+\frac{1}{2}\frac{q^2}{m^2})L-\frac{1}{4}\frac{q^2}{m^2}S)-q_\mu(L+\frac{1}{2}S)+\ldots] \\
J_{\mu\nu}(p,q) &= \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{k^2(k-q)^2((k-p)^2-m^2)} = \frac{i}{32\pi^2 m^2} \\
&\times \ [-q_\mu q_\nu(L+\frac{3}{8}S)-p_\mu p_\nu\frac{q^2}{m^2}(\frac{1}{2}L+\frac{1}{8}S) \\
&+ \ q^2\eta_{\mu\nu}(\frac{1}{4}L+\frac{1}{8}S)+(q_\mu p_\nu+q_\nu p_\mu)((\frac{1}{2}+\frac{1}{2}\frac{q^2}{m^2})L+\frac{3}{16}\frac{q^2}{m^2 S}) \\
J_{\mu\nu\alpha}(p,q) &= \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu k_\alpha}{k^2(k-q)^2((k-p)^2-m^2)} \\
&= \frac{-i}{32\pi^2 m^2}\Bigg[q_\mu q_\nu q_\alpha\left(L+\frac{5}{16}S\right)+p_\mu p_\nu p_\alpha\left(-\frac{1}{6}\frac{q^2}{m^2}\right) \\
&+ \ \big(q_\mu p_\nu p_\alpha+q_\nu p_\mu p_\alpha+q_\alpha p_\mu p_\nu\big)\left(\frac{1}{3}\frac{q^2}{m^2}L+\frac{1}{16}\frac{q^2}{m^2}S\right) \\
&+ \ \big(q_\mu q_\nu p_\alpha+q_\mu q_\alpha p_\nu+q_\nu q_\alpha p_\mu\big)\left(\left(-\frac{1}{3}-\frac{1}{2}\frac{q^2}{m^2}\right)L-\frac{5}{32}\frac{q^2}{m^2}S\right) \\
&+ \ \big(\eta_{\mu\nu}p_\alpha+\eta_{\mu\alpha}p_\nu+\eta_{\nu\alpha}p_\mu\big)\Big(\frac{1}{12}q^2L\Big) \\
&+ \ \big(\eta_{\mu\nu}q_\alpha+\eta_{\mu\alpha}q_\nu+\eta_{\nu\alpha}q_\mu\big)\Big(-\frac{1}{6}q^2L-\frac{1}{16}q^2S\Big)\Bigg]+\ldots
\end{aligned} \tag{69}
$$

for their "triangle" counterparts. Similarly higher order forms can be found, either by direct calculation or by requiring various identities which must be satisfied when the integrals are contracted with $p^\mu, q^\mu$ or with $\eta^{\mu\nu}$. Using these integral forms and substituting into Eqs. 66 and 67, one derives the results given in section 3.

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